U.G. 1st Semester Examination - 2023

MATHEMATICS

[MAJOR]

Course Code : BMTMMAJ01T

Course Title : Classical Algebra, Analytical Geometry (2D) & Calculus

[NEP-20]

Full Marks : 75 Time : 3 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **fifteen** questions: $2\times15=30$

i) Express x^4+x^2+10 as a polynomial in $(x+3)$.

ii) If $\alpha,\beta,\gamma,\delta$ be the roots of the equation $x^4+px^3+qx^2+rx+s=0$, find the value of $\sum\alpha^2\beta\gamma$.

iii) Find the equation of the straight line $\frac{x}{a}+\frac{y}{b}=2$ when the origin is shifted to the point (a, b).

iv) Find the principal value of $(1-i)^{1+i}$.

v) Determine the nature of the conic $r=\dfrac{1}{4-5\cos\theta}$ and find the length of the latus rectum.

vi) Applying Descarte's rule of sign, determine the nature of the roots of the equation $x^4+16x^2+7x-11=0$.

vii) Reduce the reciprocal equation $x^5-6x^4+7x^3+7x^2-6x+1=0$ to its standard form.

viii) If a, b, c be positive real numbers, prove that $\dfrac{a}{b}+\dfrac{b}{c}+\dfrac{c}{a}\geq 3$.

ix) Find the square root of the complex number $5-12i$.

x) Show that the equation $x^2+6xy+9y^2+4x+12y-5=0$ represents a pair of parallel lines and find the distance between them.

xi) Find the centre and radius of the circle $r=3\sin\theta+3\sqrt{3}\cos\theta$.

xii) If $xy = x^n \log x$ then show that $xy_n = (n-1)!$.

xiii) Find the pedal equation of the cardioid $r = a(1+\cos\theta)$.

xiv) Find the values of a a and f for which the curve $ax^2 + 6xy + 9y^2 + 4x + 2fy + 1 = 0$ may represent a parabola.

xv) The gradient of one of the straight lines of $ax^2 + 2hxy + by^2 = 0$ is twice that of other. Show that $8h^2 = 9ab$.

xvi) If $y = \sin 3x \cos 2x$, find y_n.

xvii) If the pair of line $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then prove that pq = –1.

xviii) Find the curvature of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at (a, 0).

xix) If $u = \sin^{-1}\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

xx) Find horizontal and vertical asymptotes of $x^2y^2 = a(x^2 + y^2)$.

xxi) Show that the sum of the intercepts of any tangent of the curve $\sqrt{x} + \sqrt{y} = \sqrt{c}$ is a constant.

xxii) Find the volume of the solid generated by the revolution of the parabola $y^2 = 4ax$ about the x-axis between the vextex and $x=a$.

xxiii) If $a > b > 0$, then show that $log\left(\frac{4}{b}\right) < log\left(\frac{1+b}{1+a}\right)$.

2. Answer any **five** questions: $5 \times 5 = 25$

i) Show that the ratio of the principal values of $(1+i)^{1-i}$ and $(1-i)^{1+i}$ is $sin(log\,2) + i\,cos(log\,2)$.

ii) If α, β, γ be the roots of $x^3 + qx + r = 0$ find the equation whose roots are $(\alpha - \beta)^2, (\beta - \gamma)^2, (\gamma - \alpha)^2$.

iii) If x, y, z are positive real numbers and $x + y + z = 1$, then prove that $8xyz \le (1-x)(1-y)(1-z) \le \frac{8}{27}$.

iv) Find the equation of the chord of the conic $\frac{l}{r} = 1 + e\cos\theta$ joining the two points whose vectorial angles $\alpha - \beta$ and $\alpha + \beta$. Hence obtain the equation of the tangent at a point whose vectorial angle α.

v) Reduce the following equation to the cannonical form and determine the type of the conic represented by the equation: $8x^2 - 12xy + 17y^2 + 16x - 12y + 3 = 0$

vi) If $I_n = \int_0^{\pi/2} x^2 \sin x dx$, n being a positive integer greater than 1, show that $I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$. Hence find value of $\int_0^{\pi/2} x^5 \sin x\, dx$. 3+2

vii) Show that the envelope of the ellipse $\frac{(x-\alpha)^2}{a^2} + \frac{(x-\beta)^2}{b^2} = 1$ where the parameters α, β are connected by the relation $\frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} = 1$ is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

b) If $y^{1/m} + y^{-1/m} = 2x$ then show that

$$(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$$

for all n.

c) If $I_n = \int_0^{\frac{\pi}{4}} tan^n \theta \, d\theta$, prove that

$n(I_{n-1} + I_{n+1}) = 1$. 2+4+4
